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QUESTION OF THE CONVERGENCE OF ITERATION METHODS OF SOLVING THE INVERSE HEAT-CONDUCTION PROBLEM

V. V. Mikhailov UDC 536.24.02

The convergence of iteration methods of solving the inverse heat-conduction problem depending on the type of desired boundary function is numerically investigated.

Iteration methods of solving inverse boundary-value heat-conduction problems (IHCP) in an extremal formulation are utilized extensively at this time. These methods are based on the search for boundary functions by starting from the requirement of minimization of a certain functional characterizing the measure of the deviation of the calculated temperatures from the temperature measured during the experiment.

Both the density of the heat flux (boundary condition (BC) of the second kind) and the temperature of the surface being heated (BC of the first kind) can be considered as the functions desired.

Fundamental attention is paid in the development of iteration methods to the construction of iterative schemes based on a search for the time dependence of the thermal flux density [1-3]. At the same time, iteration schemes based on the search for the time dependence of the surface temperature have a definite advantage since it is necessary to find a continuous function with a known value at the initial instant t = 0 (the temperature distribution over the thickness is usually known at t = 0). The thermal flux density can hence be determined by conversion of the boundary condition.

To estimate the convergence of iterative methods of solving the IHCP as a function of the kind of desired boundary function, we consider the following inverse problem in the domain $\{0 \leqslant x \leqslant b,\ 0 \leqslant t \leqslant t_p\}$:

$$C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right],\ 0 < x < b,\ 0 < t \leqslant t_p, \tag{1}$$

$$T(x,\ 0) = \varphi(x),\ 0 \leqslant x \leqslant b, \tag{2}$$

$$(2-K)\,T(0,\ t) + (1-K)\,\lambda(T)\frac{\partial T(0,\ t)}{\partial x} = u(t), \tag{3}$$

$$-\lambda(T)\frac{\partial T(b,\ t)}{\partial x} = q_2(t), \tag{4}$$

$$T(b,\ t) = f(t), \tag{5}$$

where C(T), λ(T), φ(x), q_2(t), f(t) are known functions, K is a parameter governing the type of BC on the domain boundary x = 0 (K = 1 is a BC of the first kind and K = 2 of the second kind).

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Determine the unknown function u(t) on the boundary x = 0 for a known temperature f(t) and thermal flux density $q_2(t)$ on the boundary x = b. The function u(t) is determined from the condition for the minimum of the functional

$$J[u(t)] = \frac{1}{2}\int_0^{t_p} [T(b,\ t) - f(t)]^2 dt. \tag{6}$$

Iteration approximations to the desired function are constructed by the scheme of the method of conjugate gradients

$$u_{k+1} = u_k - \alpha p_k,\ k = 0,\ 1,\ 2,\ \dots,\ p_k = -J'_{u_k} + \beta_k p_{k-1}, \tag{7}$$
$$\beta_k = (J'_{u_k} - J'_{u_{k-1}},\ J'_{u_k})/(J'_{u_{k-1}},\ J'_{u_{k-1}}),\ \beta_0 = 0,$$

where J'_u is the gradient of the functional being minimized, $u_0(t)$ is a known initial approximation, and α is the magnitude of the step in going over to the next (k + 1)-th approximation.

The gradient of the functional J'_u is calculated from formulas based on the solution of the boundary-value problem adjoint to problem (1)-(4). Following the methodology elucidated in [3], it can be shown that the expressions for the gradient of the functional (6) have the following form depending on the type of BC on the boundary x = 0:

$$J'_u = \frac{\partial}{\partial x}[A(0,\ t)\psi(0,\ t)] \quad \text{for } K = 1 \tag{8}$$

and

$$J'_u = \frac{A(0,\ t)\psi(0,\ t)}{\lambda(0,\ t)} \quad \text{for } K = 2, \tag{9}$$

while the value of the adjoint variable $\psi(0,\ t)$ is determined from the solution of the following boundary-value problem

$$-\frac{\partial\psi}{\partial t} = \frac{\partial^2}{\partial x^2}(A\psi) - \frac{\partial}{\partial x}(B\psi) + D\psi,\ \ 0 < x < b,\ 0 \leqslant t < t_p, \tag{10}$$

$$\psi(x,\ t_p) = 0,\ \ 0 \leqslant x \leqslant b, \tag{11}$$

$$\frac{A(0,\ t)\psi(0,\ t)}{\lambda(0,\ t)}\left[(2-K) + (K-1)\frac{\partial\lambda(0,\ t)}{\partial x}\right] + \left\{\frac{\partial}{\partial x}[A(0,\ t)\psi(0,\ t)] - B(0,\ t)\psi(0,\ t)\right\}(K-1) = 0, \tag{12}$$

$$\frac{A(b,\ t)\psi(b,\ t)}{\lambda(b,\ t)}\,\frac{\partial\lambda(b,\ t)}{\partial x} + \frac{\partial}{\partial x}[A(b,\ t)\psi(b,\ t)] - B(b,\ t)\psi(b,\ t) = T(b,\ t) - f(t), \tag{13}$$

where

$$A(x,\ t) = \lambda(x,\ t)/C(x,\ t);\ B(x,\ t) = 2\frac{\partial\lambda(x,\ t)}{\partial x}\Big/ C(x,\ t);$$
$$D(x,\ t) = \left(\frac{\partial^2\lambda(x,\ t)}{\partial x^2} - \frac{\partial C(x,\ t)}{\partial t}\right)\Big/ C(x,\ t).$$

The dependences A(x, t), B(x, t), D(x, t) as well as $\lambda(0,\ t)$, $\partial\lambda(0,\ t)/\partial x$, $\lambda(b,\ t)$ and $\partial\lambda(b,\ t)/\partial x$ are determined from the solution of problem (1)-(4).

As has been noted in [1], there is no uniform convergence in the calculation of the gradient by means of (9), and the accuracy of the solution of IHCP depends to a considerable extent on the selection of the initial approximation $u_0(t)$. In this connection, to assure uniform convergence in the case K = 2 the iteration sequence can be constructed by using

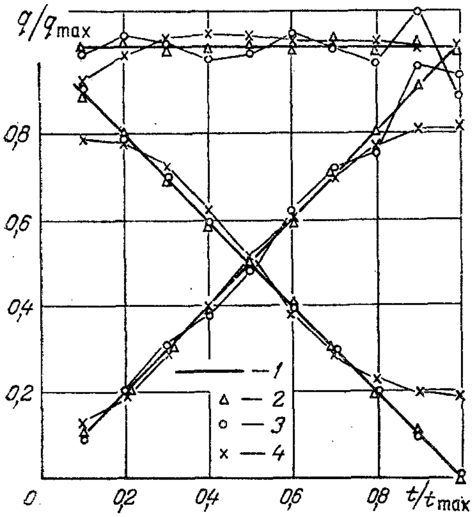

Fig. 1. Restoration of different time dependences of the thermal flux density on one of the boundaries by means of the temperature on the other heat-insulated boundary: 1) true values; 2) using (8); 3) using (9); 4) using (14).

an expression for the gradient with respect to the derivative of the heat flux density with respect to time $\dot{u} = du/dt$ [2, 3]:

$$J'_{\dot{u}} = \int_{t}^{t_p} \frac{A(0, \tau)\psi(0, \tau)}{\lambda(0, \tau)} d\tau. \tag{14}$$

The magnitude of the step α in going from $u_k(t)$ to $u_{k+1}(t)$ is determined from the condition $\min\limits_{\alpha} J(u_k - \alpha p_k)$.

A calculation algorithm has been developed on the basis of the above, and a number of inverse heat-conduction problems have been solved, including the restoration of different time dependences of the thermal flux density on the boundary of a flat plate x = 0 for a given temperature on the other heat-insulated boundary x = b. Here the heat flux density was estimated in the case K = 1 (search for the surface temperature) by an approximate formula of second order accuracy.

Results of the computations are displayed in Fig. 1. The dependences presented are obtained for the very same number of iterations, which equals the number of parameters for a discrete representation of the desired dependences for all the computed cases.

The results obtained showed that the iteration scheme based on searching for the surface temperature (computation of the gradient by means of (8)) is not only uniformly convergent but also assures the greatest accuracy in restoring the heat flux density, other conditions being equal, as compared to schemes using (9) or (10).

NOTATION

t, time; t_p, length of the time interval; x, space coordinate; b, thickness; T(x, t), temperature; C(T), bulk specific heat of the material; λ(T), heat conduction coefficient of the material; φ(x), initial temperature distribution; q, thermal flux density; f(t), measured temperature; ψ(x, t), adjoint variable; α, β, p, parameters of the method of conjugate gradients; and k, number of the iteration.

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AN ITERATION ALGORITHM FOR THE SOLUTION OF THE INVERSE BOUNDARY-VALUE PROBLEM OF HEAT CONDUCTION

S. V. Pavlyuk UDC 536.24

An iterative procedure is constructed for solving the inverse boundary-value problem of heat conductivity in an extremal formulation on the basis of solving a Cauchy problem.

Following [1, 2], the solution of the nonstationary heat-conduction problem

$$\bar{C}_v(T)\frac{\partial T}{\partial \tau} = \bar{\lambda}(T)\frac{\partial^2 T}{\partial X^2} + \bar{\lambda}'(T)\left(\frac{\partial T}{\partial X}\right)^2, \tag{1}$$

$$T|_{\tau=0} = 0, \tag{2}$$

$$\left.\frac{\partial T}{\partial X}\right|_{X=0} = 0,\ T|_{X=1} = T_w(\tau), \tag{3}$$

where $\tau = a_0 t/R^2$, $X = x/R$, R are the dimensionless time, coordinate, and characteristic linear dimension, respectively, and $\bar{C}_v = C_v/C_{v,0}$, $\bar{\lambda} = \lambda/\lambda_0$ are the relative values of the bulk specific heat and the heat-conduction coefficient, is written in the form

$$T(X,\ \tau) = \lim_{N\to\infty}\left[\sum_{n=0}^{N-1}\Omega_n(X,\ Y_1)\,Y_{n+1}(\tau) + W(X,\ \mathbf{Y})\right], \tag{4}$$

$$\Omega_n(X,\ Y_1) = X^{2n}\left[\frac{\bar{C}_v(Y_1)}{\bar{\lambda}(Y_1)}\right]^n,$$

where the vector function $\mathbf{Y} = \{Y_1,\ Y_2,\ \ldots,\ Y_N\}$ is defined as the solution of the Cauchy problem

$$\left.\begin{aligned}&\frac{dY_n}{d\tau} = \varepsilon_n Y_{n+1},\ n = 1,\ 2,\ \ldots,\ N-1,\\ &\frac{dY_N}{d\tau} = \frac{\varepsilon_N}{\Omega_N(X, Y_1)}\left[T_w(\tau) - W(1,\ \mathbf{Y}) - \sum_{n=0}^{N-1}\Omega_n(1,\ Y_1)\,Y_{n+1}(\tau)\right],\\ &Y_n|_{\tau=0} = 0.\end{aligned}\right\} \tag{5}$$

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